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ABSTRACT

The paper examines a longitudinal model of a manpower system in which the demands for effective manpower are determined by the state of a finite Markov chain, and there are delays in training an adequate supply of effective manpower. An operational method of calculating optimal appointment policies is presented. This calculation can in turn be used to find the equilibrium operating rules for the system. The model is a useful device for measuring the impact of alternate assumptions about continuation rates, manpower utilization policies, the demands, and the transition probabilities in the demand process. (Author)

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MANPOWER PLANNING WITH UNCERTAIN REQUIREMENTS

by

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13. ABSTRACT

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ABSTRACT

This paper examines a longitudinal model of a manpower system in which the demands for effective manpower is determined by the state of a Fiscal Nation check, and there are delays in training an adequate supply of effective manpower. We present an operational method of calculating optimal appointment policies. This calculation can in turn be used to find the equilibrium operating rules for the system. The model is a useful device for measuring the impact of alternate assumptions about continuation rates, manpower utilization policies, the demands, and the transition probabilities in the demand process.

1. INTRODUCTION

This paper considers the problem of providing an adequate supply of effective area weapons when biased fire weapons in accuracy are used. The use of biased fire weapons in the training of effective personnel.

The model described in this paper is useful in three specific ways. (1) It can calculate optimal weapons input policy for planning against objectives, (2) it can be used to evaluate the effects of alternate training, maintenance, and refitting policies on system performance, and finally (3) it can measure the sensitivity of optimal policies and system performance to various assumptions on the nature of the non-linearity aspects of the search process.

In Section 2 we present a model for the fire of personnel. This model attempts to capture the essential features of the weapons system in a simple framework by utilizing the beta-binomial probability of various weapons effects. The simultaneous nature of the search process is described in Section 3. We view λ_i, the demand for weapon i to be determined by the users of a training system. In Section 4 we present two alternate command objectives. The first objective is to minimize the expected, weighted, discounted square of the error between the effective supply and demand for weapons. The second objective is to minimize the expected discounted square error between the actual weapons counts and ideal counts for any desired levels. In Section 5 we describe the type of the optimal policy and optimal value functions. Numerical solutions are presented for the two control objectives. In Section 6 we outline several uses for the model. The first is the determination of the expected weapons counts when the optimal apportionment policy is used. Various levels of the equilibrium counts can then be used to calculate the expected cost of the apportionment policy along with such expected measure of system performance. These calculations allow us to measure the tradeoff between cost and performance for alternate weapons policies. In Section 7 we also

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Utilization

Demand \ Length of Service	1	2	3	4	5	6	7	8
1	~0.5	~0.5	1	1	0.8	0.8	0.2	0.2
2	~0.5	~0.5	1	1	1	0.8	0.3	0.2
3	~0.5	~0.25	1	1	1	1	0.8	0.8
4	~0.5	~0.3	1	1	1	1	0.8	0.8
5	~0.5	~0.3	1	1	1	1	0.8	0.8
6	~0.5	~0.3	1	1	1	1	0.8	0.8

3. THE CONTROL OBJECTIVE

We will formulate two quadratic objective functions for controlling the flow of manpower through the system. We selected quadratic objectives for two reasons. First, they lead to a model for which results can be readily calculated. Second, the objectives are a reasonable measure of the departure of actual system performance from an ideal performance.

The simplest control objective is to minimize the expected weighted squared error between the supply and demand for effective manpower. Let w_j for $j = 1,2, \ldots, n$ be positive weights that sum to 1. If $i(t) = i$, and $s(t)$, then the expected error at time $t + 1$, as a function of $f = f(t + 1)$, is

$$(1) \qquad E^i[s(t),f(t+1)] = \sum_{j=1}^{n} r_{ij} w_j \left(z_j - f(t+1) - \sum_{k=1}^{m-1} q_{ik} s_k(t) \right)^2$$

Our second control objective is to provide a smooth flow of manpower through the system and to come as close as possible to meeting the demand for effective manpower. If the manpower system was always in state j, then an ideal distribution would satisfy

$$(2) \qquad \sum_{k=1}^{m} d_{jk} s_k = z_j ,$$

and

$$s_{k+1} = \left\{ \begin{array}{ll} f & \text{if} \quad k = 0 \\ q_{jk} s_k & \text{for} \quad k = 1,2, \ldots, m-1 \end{array} \right\}.$$

Note that (2) can be rewritten as

$$(3) \qquad s_{k+1} = \left\{ \begin{array}{ll} f & \text{if} \quad k = 0 \\ \left[\prod_{\ell}^{k} q_{j\ell} \right] f & \text{for} \quad k = 1,2, \ldots, m-1 \end{array} \right\}.$$

For $k = 2,3, \ldots, m$ define

$$P_{jk} = \prod_{\ell=1}^{k-1} q_{jk} \tag{4}$$

and $P_{j1} = 1$.

Combining (4), (3), and (2) we obtain

$$s_{jk} = (z_j P_{jk}) / \sum_{k=1}^{m} P_{jk} d_{jk} \tag{5}$$

The discrepancy between actual and ideal is measured by

$$\sum_{k=1}^{m} (s_k(t+1) - s_{jk})^2 \tag{6}$$

Using the law of motion (II;3) we can rewrite (7) as

$$E^{ij}[s(t), f(t+1)] = (f(t+1) - s_{j1})^2 + \sum_{k=1}^{m-1} (q_{ik} s_k(t) - s_{jk+1})^2 \tag{8}$$

This is a measure of the discrepancy at time $t + 1$ conditioned on four facts; manpower $s(t)$ at time t, demand i at time t, accessions $f(t + 1)$ in period $t + 1$, and demand j at time $t + 1$. The function is quadratic, strictly convex in $f(t + 1)$ and convex in $s(t)$. The expected discrepancy is

$$E^{i}[s(t), f(t+1)] = \sum_{j=1}^{n} r_{ij} E^{ij}[s(t), f(t+1)] \tag{9}$$

is also quadratic convex in f (strictly) and s.

We note that it is possible to weigh the functions $E^{w}[s(t), f(t + 1)]$ in order to place greater or less importance on the expected discrepancy from

state j .

For either objective in a T stage problem, where $s(t)$, and $f(t)$ are the observed values of the manpowerr stocks and accessions for $t = 1,2, \ldots, T$, and $i(t)$ is the state of demand, the total discounted error is

$$(10) \qquad \sum_{t=1}^{T} \delta^{t-1} E^{i(t)}[s(t),f(t)]$$

where $0 < \delta < 1$ is a discount facctor. The objective is to minimize the expected value of (10) over all possible appointment sequences.

4. THE OPTIMAL APPOINTMENT POLICY

This section presents two examples of optimal appointment policies: one for each of the objectives presented in Section 3.

A theoretical description of the optimization procedure is presented in the appendix. We shall use three important results from the appendix:

(i) For any finite planning horizon T, and any demand state i, there is an optimal appointment policy that is a linear function of the manpower stock s. For our model

$$a = b_0^{i,T} + \sum_{k=1}^{n} b_k^{i,T} s_k .$$

(ii) For any finite planning horizon T, and any demand state i, the minimum expected error is a convex quadratic function of the manpower stocks s:

$$s' P^{i,T} s + q^{i,T} s + r^{i,T}$$

(iii) As the planning horizon T becomes large, the values of $b^{i,T}$, $P^{i,T}$, $q^{i,T}$, $r^{i,T}$, converge to b^i, P^i, q^i, r^i.

In the examples we have solved to date, the convergence has been quite rapid. For the first example below a 1% change was observed between a 20 and 21 period horizon. In the second example convergence was nearly perfect in 5 periods.

The first solution below uses the objective (3:1) with discount factor $\delta = 0.95$ and weights

$$w_j = \begin{cases} 1/18 & \text{if } j = 1,2 \\ 4/18 & \text{if } j = 3,4,5,6 \end{cases} .$$

The optimal appointment policy is shown in Table 1.

For the second control objective, (3:9), there is a considerable simplification in the optimal decision rule: $f = k_0^x + \sum_{k=1}^{n} k^x k_{ak}$. In the second case $k_0^x = 0$ for $k \geq 1$. Thus $f = k_0^x$ independently of the current manpower stock s.

The optimal input as a function of t is shown in Table 2. In this calculation we set $\delta = 0.95$ and did not weigh the various terms in the sum (3:9).

TABLE 1

0	0.67	-.05	-.17	-.14	.01	.30	0.0	.10	0
1	0.04	-.05	-.16	-.13	.07	.10	.01	.10	0
2	0.01	-.07	.06	.00	.17	.21	.10	.09	0
3	0.05	-.01	0.0	.01	.08	.07	.05	.06	0
4	0.45	-.01	0.00	.00	.06	.04	.00	.02	0
5	0.71	-.06	-.03	-.01	.06	.00	.00	.01	0

TABLE 2

0	0.0
1	0.0
2	0.0
3	0.0
4	0.0
5	0.0

[illegible]

[illegible] [illegible] [illegible] [

(45)

$$[illegible]$$

(46)

Where $[illegible]$ is the exchange integral between $[illegible]$ and $[illegible]$ and $[illegible]$ is the overlap integral between $[illegible]$ and $[illegible]$. The $[illegible]$ is the Coulomb integral between $[illegible]$ and $[illegible]$. The $[illegible]$ is the Coulomb integral between $[illegible]$ and $[illegible]$. The $[illegible]$ is the Coulomb integral between $[illegible]$ and $[illegible]$. The $[illegible]$ is the Coulomb integral between $[illegible]$ and $[illegible]$.

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APPENDIX

DISCRETE TIME, LINEAR-QUADRATIC, DYNAMIC PROGRAMS WITH MARKOVIAN PARAMETERS

This appendix presents, in a more general form, the theoretical results used in the paper. We employ a notation close to that of the text.

The state of the system with $T+1$ periods remaining is (s,i). Given action f, the state of the system with T periods remaining will be $(D^i(s,f),j)$ with probability r_{ij}, where, for each i, $D^i(s,f)$ is a linear function of s and f.

The error associated with choosing f in state (s,i) is $E^i(s,f)$, where for each i $E^i(s,f)$ is quadratic and convex in s and strictly convex in f. Future errors are discounted at rate δ.

For $i = 1,2, \ldots, n$ define $V_T(s,i)$ to be the minimum expected discounted error for a T period problem that starts with manpower stocks s, and demand state i.

For any choice of f, we have

$$V_{T+1}(s,i) \leq E^i(s,f) + \delta \sum_{j=1}^{n} r_{ij} V_T(D^i(s,f),j) \tag{1}$$

By the usual principal of optimality arguments we can establish that the functions $V_{T+1}(s,i)$ must satisfy the following functional equation.

$$V_{T+1}(s,i) = \min_f \left[E^i(s,f) + \delta \sum_{j=1}^{n} r_{ij} V_T(D^i(s,f),j) \right] \tag{2}$$

where $V_0(s,i) = 0$ for each s and i.

Now assume, that $V_T(s,i)$ is a convex quadratic function of s for each i. Thus we can write $V_T(s,i) = s'P^{i,T}s + q^{i,T}s + r^{i,T}$ where $P^{i,T}$ is positive semi-definite, and symmetric. It is easy to show that $V_T(D^i(s,f),j)$ is a convex quadratic function of (s,f), and therefore that

(3) $$E^i(s,f) + \delta \sum_{j=1}^{n} r_{ij} V_T(D^i(s,f),j)$$

is quadratic, convex in s, and strictly convex in f.

When the quadratic function (3) is differentiated with respect to f, then a linear equation results, and the unique f that minimizes (3) as a function of s is in the form

(4) $$f_{T+1}(s,i) = B^{i,T+1}(s)$$

where for each i and $T+1$, $B^{i,T+1}(s)$ is linear in s. When the optimal policy (4) is substituted into (3) we obtain a convex quadratic expression for $V_{T+1}(s,i)$.

Thus for any finite horizon T the optimal value and policy functions are, respectively, linear and quadratic. We now turn to the question of convergence of policies as the planning horizon increases.

Let $V_T(s,i) = s'P^{i,T}s + q^{i,T}s + r^{i,T}$ be the optimal value function starting in state i for an ℓ period problem.

When the discount factor δ is strictly less than 1, then the sequence $V^T(s,i)$ will remain bounded if there exists a policy $f(s,i)$ such that this policy leads to a bounded sequence $s(t)$ for any initial $s(o)$ and any evolution of the demand sequence $i(t)$. In the model described in this paper, the policy $f(s,i) = 0$ will insure $s(t) = 0$ for $t > m$. The general question of convergence of $V^T(s,i)$ and $B^T(s,i)$ is difficult, and is currently the object of further investigation by the author.